The ECHO code: from classical MHD to GRMHD in dynamical spacetimes

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The ECHO code

- **Eulerian Conservative High Order code**: the aim is to combine shock-capturing properties and accuracy for small scale wave propagation and turbulence, in a 3+1 approach
- Modular structure, F90 language, MPI parallelization
- Many physical modules (MHD, RMHD, GRMHD, GRMD,…)
- Any metric allowed (1-,2- or 3-D), even time-dependent
- Finite-difference scheme, Runge-Kutta time-stepping
- UCT strategy for the magnetic field (staggered grid, 4-state B fluxes)
- High-order reconstruction procedures (explicit and implicit)
- Central-type Riemann solvers (LLF, HLL, HLLC)
- Upgrades: resistivity, radiation hydrodynamics, evolving spacetimes
2-D MHD compressible tearing mode

- Resistivity added to the MHD module
- Compact 7th order schemes are best suited to simulate the tearing mode (256x128, R=5000, δ=0.1, Va=1, β=5)
- Linear growth rates reproduced and magnetic islands coalescence
3-D MHD compressible tearing mode

- Additional instabilities and onset of turbulence are found in 3-D MHD simulations of the compressible tearing mode, with application to solar wind (Landi et al. 2008; Bettarini et al. 2009)
MHD Alfvénic pulses in 2-D coronal loops

- After large flares, coronal loops show transverse, quickly damped oscillations. High shear viscosity/resistivity?
- Spreading of ideal MHD Alfvénic pulses propagating up and down the loop show the same behavior (Del Zanna et al. 2005)
Parametric instabilities of Alfvén waves

- Alfvén waves in the (fast) solar wind evolve with heliocentric distance.
- Parametric decay is an instability expected in compressible MHD.
- MHD simulations in multi-D \((Del Zanna et al. 2001, Del Zanna 2001)\)
- Radial expansion effects with the expanding box model \((Velli et al. 1992)\), adopting a Cartesian-like time-dependent metric \((Del Zanna et al. 2011)\)
The EBM in ECHO

To mimic the modifications to transverse gradients induced by the radial expansion (at constant solar wind speed), in EBM we define a scale factor

\[ a(t) = \frac{R(t)}{R_0} = 1 + \epsilon t, \quad \epsilon = \frac{V_{SW}}{R_0}. \]

EBM in ECHO is implemented by simply prescribing

\[ \gamma_{ij} = \text{diag}(1, a^2, a^2) \Rightarrow \gamma^{1/2} = a^2, \]

recalling that, e.g. \( v_1 = v^1 = v_x, \ v_2 = a^2 v^2 = a v_y, \ v_3 = a^2 v^3 = a v_z. \)
RMHD model of the Crab Nebula

- Jet-torus structure reproduced, velocities in the right range, variability
- Diagnostics: synchrotron and IC non-thermal emission
  - Del Zanna et al. 2004, 2006; Volpi et al. 2008

![Diagram showing jet, torus, counter-jet, inner ring, and knot]
RMHD model of the Crab Nebula

- Jet-torus structure predicted in $\gamma$-rays as well
- Shrinkage of PWN size with increasing frequency
- Crab Nebula’s complete spectrum basically reproduced by using two families of particles: relic and accelerated electrons
GRMHD: Eulerian 3+1 approach

- Set of 8 conservation laws + 1 constraint:

\[
\partial_i (\sqrt{\gamma} D) + \partial_i [\sqrt{\gamma} (\alpha \nu^i - \beta^i) D] = 0
\]
\[
\partial_i (\sqrt{\gamma} S_j) + \partial_i [\sqrt{\gamma} (\alpha W^i_j - \beta^i S_j)] = \sqrt{\gamma} (\alpha W^k \partial_j \gamma_{ik} / 2 + S_i \partial_j \beta^i - U \partial_j \alpha)
\]
\[
\partial_i (\sqrt{\gamma} U) + \partial_i [\sqrt{\gamma} (\alpha S^i - \beta^i U)] = \sqrt{\gamma} (\alpha K \gamma W^j - S^i \partial_i \alpha)
\]
\[
\partial_i (\sqrt{\gamma} B^i) + \partial_i [\sqrt{\gamma} (\alpha \nu^i - \beta^i) B^i - \sqrt{\gamma} (\alpha \nu^i - \beta^j) B^j] = 0; \quad \partial_i (\sqrt{\gamma} B^i) = 0
\]

- No Lie derivatives nor Christoffel symbols needed in source terms
- The lapse function \( \alpha \), shift vector \( \beta \), metric tensor \( \gamma \) and the extrinsic curvature \( K \) may be time-dependent (evolved through Einstein’s eqs.)
- Only familiar spatial 3-D vectors and tensors, easy RMHD and MHD limits

\[
D = \rho \Gamma; \quad \tilde{S} = \rho h \Gamma^2 \tilde{\nu} + \tilde{E} \times \tilde{B}; \quad U = \rho h \Gamma^2 - p + (E^2 + B^2) / 2
\]
\[
\tilde{W} = \rho h \Gamma^2 \tilde{\nu} \tilde{\nu} + p \tilde{\gamma} - \tilde{E} \tilde{E} - \tilde{B} \tilde{B} + (E^2 + B^2) / 2 \tilde{\gamma}; \quad \tilde{E} = -\tilde{\nu} \times \tilde{B}
\]
Selected tests

RMHD CP Alfvén wave: convergence

Magnetized torus in Kerr
Accretion disks of SMBHs

- The circumbinary disk in the post-merger phase of SMBHs is affected by mass loss and recoil of the resulting SMBH. 2-D GRHD simulations of kicked disks investigate this scenario (Zanotti et al. 2010)
- Radiation-hydrodynamics module added to ECHO. Bondi-Hoyle accretion studied in the presence of radiation (Zanotti et al. 2011)
Proto-magnetars and GRB jets

- Long duration GRBs could be generated by proto-magnetar winds collimating polar relativistic jets which finally escape from the stellar progenitor.
- Same magnetic pinching effect due to toroidal fields as in PWNe?
- Full magnetized wind + jet evolution (Bucciantini et al. 2009). From NS surface to jet propagating out of the star (10s of physical time).
Towards collapse to proto-magnetars?

- Our ultimate goal would be to simulate the formation of a proto-NS (or proto-magnetar), needed to power long GRB jets.
- Core collapse: MRI and dynamo effects? (Cerda-Duran et al. 2008)
- Requirements (as in the CoCoNuT code, Dimmelmeier et al. 2002...):
  - Full GR approach: solution of Einstein’s equations
  - Hydrodynamics and GRMHD
  - Realistic microphysics (EoS), deleptonization (neutrino transport)
  - Spherical coordinates and axisymmetry
- Recent progress: X-ECHO (Bucciantini & Del Zanna, A&A 528, A101, 2011)
  - Based on the eXtended CFC approximation (Cordero-Carrion et al. 2009)
  - Elliptical Einstein eqs., hyperbolic GRMHD eqs.
  - No gravitational waves (asymptotic flatness)
  - Work in progress: pulsations of magnetized NS
From CFC to XCFC

- 3+1 decomposition, ADM formalism
- Fully constrained scheme for Einstein’s eqs. (FCF: Bonazzola et al. 2004)
  - conformal flatness (conformal factor \( \psi \))
  - maximum slicing (K=0)
  - conformal decomposition of extrinsic curvature
  - widely used for core collapse, NS stability and evolution
  - axisymmetry: negligible deviations from full GR, exact in 1-D
- From CFC to XCFC (Cordero-Carrion et al. 2009):
  - 8 equations and 8 unknown functions (5 in CFC)
  - full decoupling: hierarchical solution of Poisson-like eqs.
  - self-consistent update of metric and primitive variables
  - local uniqueness of solution
The XCFC equations

\[ ds^2 := g_{\mu \nu} \, dx^\mu \, dx^\nu = -\alpha^2 \, dt^2 + \gamma_{ij} \left( dx^i + \beta^i \, dt \right) \left( dx^j + \beta^j \, dt \right), \quad \gamma_{ij} := \psi^4 \, f_{ij}, \]

\[ \Delta_L W^i = 8\pi f^{ij} \hat{S}_j, \]
\[ \Delta \psi = -2\pi \hat{E} \, \psi^{-1} - \frac{1}{8} f_{ik} f_{jl} \hat{A}^{kl} \hat{A}_{ij} \psi^{-7}, \]
\[ \Delta (\alpha \psi) = \left[ 2\pi \left( \hat{E} + 2\hat{S} \right) \psi^{-2} + \frac{7}{8} f_{ik} f_{jl} \hat{A}^{kl} \hat{A}_{ij} \psi^{-8} \right] \alpha \psi, \]
\[ \Delta_L \beta^i = 16\pi \alpha \psi^{-6} f^{ij} \hat{S}_j + 2\hat{A}^{ij} \nabla_j (\alpha \psi^{-6}), \]

\[ \Delta_L \beta^i := \nabla_j (L \beta)^{ij} = \Delta \beta^i + \frac{1}{3} \nabla^i \left( \nabla_j \beta^j \right), \]

\[ \hat{S}_j := \psi^6 S_j, \quad \hat{E} := \psi^6 E, \quad \hat{S} := \psi^6 S, \]
\[ \hat{A}^{ij} = \nabla^i W^j + \nabla^j W^i - \frac{2}{3} \left( \nabla_k W^k \right) f^{ij}. \]
X-ECHO: numerical approach

- Spherical coordinates for background metric and axisymmetry
- Mixed spectral/direct inversion techniques for the metric solver:
  - Decomposition in spherical harmonics of Poisson-like eqs.
  - Gaussian quadrature in polar angle for source terms
  - Finite-differences (second order) and tri-diagonal matrix inversion in $r$
- GRMHD eqs. in conservative form treated as in ECHO
  - UCT for magnetic field
  - Upwind reconstruction (usually MC limiter)
  - Simplified Riemann solver (HLL or HLLC)
- XNS solver (in F90) for exact NS polytropic equilibria in quasi-isotropic coordinates (small deviations expected in CFC)
  - Uniformly or differentially rotating stars
  - Polytropic-type purely toroidal magnetic field
X-ECHO: selected tests

NS migration to stable branch (1D)

NS equilibrium and vibrations (2D)
X-ECHO: selected tests

NS collapse to BH (1D and rotating 2D)
X-ECHO: selected tests

Magnetized NS equilibrium (2D)

Magnetized NS collapse (2D)
Conclusions

- **ECHO** is an Eulerian scheme for conservation laws in 3+1
- Numerical recipes: finite-differences, UCT, HLL, high-order
- System-independent routines, modular structure, F90, MPI
- Wide choice of physics and geometry modules
- Full 3+1 formalism, possible coupling with any Einstein solver
- Coupling with XCFC solver: X-ECHO
- Several different applications (not much GR so far!):
  - Compressible 2-D and 3-D MHD resistive/shear instabilities
  - Alfvén waves propagation and evolution in corona and solar wind
  - RMHD model for Pulsar Wind Nebulae: dynamics and emission
  - Proto-NS and magnetar GRMHD winds, GRB jets
  - Accretion onto SMBHs, even with radiation

Thank you!