Dynamical Supersymmetry Breaking and Phase Diagram of the Lattice Wess-Zumino Model

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Motivation

An important issue in the study of supersymmetric models is the occurence of non-perturbative dynamical supersymmetry breaking.

The problem can be studied in $1+1$ dimensional lattice models where numerical tools are more efficient and should be easier to obtain definite answer to the relevant questions.

A simple theoretical laboratory is the $N=1$ Wess-Zumino model that does not involve gauge fields.

Since S^3B is closely related to the symmetry properties of the ground state, it appears to be reasonable to adopt some kind of Hamiltonian formulation.

Outline

- The model and its lattice Hamiltonian formulation
- Pattern of supersymmetry breaking using different prepotential: a cubic prepotential and a quadratic one
- Discussions of the numerical results using two different methods

General remarks

Let us remind the (continuum) $N = 1$ supersymmetry algebra

$$\{Q_\alpha, Q_\beta\} = 2(\not{P} C)_{\alpha\beta}$$

since P_i are not conserved on the lattice, a lattice formulation of a supersymmetric model must break this eq. explicitly.

A very important advantage of the Hamiltonian formulation is the possibility of conserving exactly a key subalgebra of this eq. [Elitzur, Rabinovici, Schwimmer ,1982]; specializing to $1 + 1$ dimensions, in a Majorana basis $\gamma_0 = C = \sigma_2$, $\gamma_1 = i\sigma_3$, the eq. becomes

$$Q_1^2 = Q_2^2 = P^0 \equiv H, \qquad \{Q_1, Q_2\} = 2P^1 \equiv 2P,$$

On the lattice, since H is conserved but P is not, we can pick up one of the supercharges, say, Q_1, build a discretized version Q_L and define the lattice Hamiltonian to be $H = Q_L^2$.

Notice that $Q_1^2 = H$ is enough to guarantee that $E_0 \geq 0$.

Wess-Zumino model

The continuum 2-dimensional Wess-Zumino model is defined by the SUSY generators

$$Q_{1,2} = \int dx \left[p(x)\psi_{1,2}(x) - \left(\frac{\partial\varphi}{\partial x} \pm V(\varphi(x))\right)\psi_{2,1}(x) \right],$$

where $\varphi(x)$ is a real scalar field and $\psi(x)$ is a Majorana fermion.

The discretized supercharge with arbitrary $V(\varphi)$ [Elitzur, Rabinovici, Schwimmer ,1982; Ranft, Schiller, 1984]

$$Q_L = \sum_{n=1}^{L} \left[p_n \psi_{1,n} - \left(\frac{\varphi_{n+1} - \varphi_{n-1}}{2} + V(\varphi_n) \right) \psi_{2,n} \right].$$

and the Hamiltonian takes the form

$$H = Q^2 = \frac{1}{2} \sum_{n=1}^{L} \left[\pi_n^2 + \left(\frac{\phi_{n+1} - \phi_{n-1}}{2} + V(\phi_n) \right)^2 - (\chi_n^\dagger \chi_{n+1} + h.c.) + (-1)^n V'(\phi_n) \left(2\chi_n^\dagger \chi_n - 1 \right) \right]$$

Supersymmetry Breaking

The problem of predicting the pattern of supersymmetry breaking is not easy. In principle, the form of $V(\varphi(x))$ is enough to determine whether supersymmetry is broken or not. At least at tree level SUSY is broken if and only if V has no zeros.

The Witten index can help in the analysis. If $V(\varphi)$ has an odd number of zeroes then $I \neq 0$ and SUSY is unbroken. If $V(\varphi)$ has an even number of zeroes, when $I = 0$ we can not conclude anything.

An alternative non-perturbative analysis for $I = 0$ is welcome. The simplest way to analyze the pattern of SUSY breaking for a given V is to compute the ground state energy $E_0 \Longrightarrow$ numerical simulations and/or strong coupling.

Predictions for the Model

- All the results for the model with cubic prepotential, $V = \varphi^3$, indicated unbroken supersymmetry.

- Dynamical supersymmetry breaking in the model with quadratic prepotential

$$V = \lambda_2 \varphi^2 + \lambda_0$$

  along a line of constant λ_2 predicts the existence of two phases:

  - a phase of broken SUSY with unbroken discrete Z_2 at high λ_0 and
  - a phase of unbroken SUSY with broken Z_2 at low λ_0, separated by a single phase transition.

Strong coupling analysis of SUSY breaking

For polynomial $V(\varphi)$, the relevant parameter is just its degree q.

- For odd q, strong coupling and weak coupling expansion results agree and supersymmetry is expected to be unbroken.

  This conclusion gains further support from the nonvanishing value of the Witten index.

- For even q in strong coupling, the ground state has a positive energy density also for $L \to \infty$ and supersymmetry appears to be broken. In particular, for $V = \lambda_2 \varphi^2 + \lambda_0$, weak coupling predicts unbroken SUSY for $\lambda_0 < 0$, whereas strong coupling prediction gives broken SUSY for all λ_0.

Numerical simulation of the model seems to be the only way to answer the question of symmetry breaking.

The basic ingredient of the GFMC is that project a generic state $|i>$over the ground state

$$|\Psi_0>= lim_{t\to\infty} exp(-tH)|i>$$

and the ground state energy is obtained by

$$E_0 =< \Psi_0|H|\Psi_0>$$

Numerical results of the first method: Odd prepotential

The ground-state energy density E_0/L vs. $1/K$ at $V = \varphi^3$, $L = 22$, with statistics of 1 M iterations for $K < 5000$, 500 k iterations at $K = 5000$, and 300 k iterations at $K = 10000$. (Using the GFMC algorithm).

More interesting case: Even prepotential

When $V = \lambda_2 \varphi^2 + \lambda_0$

For fixed $\lambda_2 = 0.5$, we may expect (in the $L \to \infty$ limit) a phase transition at $\lambda_0 = \lambda_0^{(c)}(\lambda_2)$

separating a phase of broken SUSY and unbroken Z_2 (high λ_0) from a phase of unbroken SUSY and unbroken Z_2 (low λ_0).

Even prepotential

The ground-state energy density E_0/L vs. λ_0 The solid line is a fit to the form $E_0/L =$

$a\sqrt{\lambda_0 - \lambda_0^{(c)}}$. These data gives $\lambda_0^{(c)} \approx -0.53$.

Binder cumulant

The usual technique for the study of a phase transition is the crossing method applied to the Binder cumulant, B. The crossing method consists in plotting B vs. λ_0 for several values of L. The crossing point $\lambda_0^{\rm cr}(L_1, L_2)$, determined by the condition

$$B(\lambda_0^{\rm cr}, L_1) = B(\lambda_0^{\rm cr}, L_2)$$

is an estimator of $\lambda_0^{(c)}$.

The convergence is dominated by the critical exponent ν of the correlation length and by the critical exponent ω of the leading corrections to scaling [Pelissetto - Vicari, 2002]

$$\lambda_0^{\rm cr}(L_1, L_2) = \lambda_0^{(c)} + O(L_1^{-\omega-1/\nu}, L_2^{-\omega-1/\nu})$$

we expect the phase transition we are studying to be in the Ising universality class, for which $\nu = 1$ and $\omega = 2$, and therefore we expect fast convergence $\lambda_0^{\rm cr} \to \lambda_0^{(c)}$.

The Binder cumulant B vs. λ_0. $(\lambda_0^{(c)} = -0.48 \pm 0.01)$

Numerical results of the second method

The method is based on the calculation of rigorous lower bounds on the ground state energy density in the *infinite-lattice* limit. Such bounds are useful in the discussion of the SUSY breaking as follows

The lattice version of the Wess-Zumino model conserves enough supersymmetry to prove that the ground state has a non negative energy density $\rho \geq 0$, as its continuum limit.

Moreover the ground state is supersymmetric if and only if $\rho = 0$, whereas it breaks (dynamically) supersymmetry if $\rho > 0$. Therefore, if an exact positive lower bound ρ_{LB} is found with $0 < \rho_{\mathrm{LB}} \leq \rho$, we can claim that supersymmetry is broken.

The idea is to construct a sequence $\rho^{(L)}$ of exact lower bounds representing the ground state energy densities of modified lattice Hamiltonians describing a cluster of L sites and converging to $\rho^{(L)} \to \rho$ in the limit $L \to \infty$. The bounds $\rho^{(L)}$ can be computed numerically on a finite lattice with L sites.

The relevant quantity for our analysis is the ground state energy density ρ evaluated on the infinite lattice

$$\rho = \lim_{L\to\infty} \frac{E_0(L)}{L}$$

It can be used to tell between the two phases of the model: supersymmetric with $\rho = 0$ or broken with $\rho > 0$.

Derivation of the bounds

Given a translation-invariant Hamiltonian H on the lattice Z it is possible to obtain a lower bounds on its ground state density from a cluster decomposition of H.

i.e., give a suitable finite sublattice Λ into Λ', it is possible to introduce a modified Hamiltonian $\tilde{H}$ restricted to Λ such that its energy density ρ_Λ bounds ρ from below. The difference between H and $\tilde{H}$ amounts to a simple rescaling of its coupling constants. [Anderson, 71]

The only restriction on H being that could include nearest-neighbor interactions terms, next-to-nearest-neighbor coupling, and so on until some finite distance.

- We compute numerically $\rho^{(L)}$ at various values of the cluster L
- If we find $\rho^{(L)} > 0$ for some L we conclude that we are in the broken phase.
- We know that $\rho^{(L)} \to \rho$ for $L \to \infty$ and the study of $\rho^{(L)}$ as a function of *both* L and the coupling constants permit the identification of the phase in all cases.
- The calculation of $\rho^{(L)}$ is numerically feasible because does not require the computation of an infinite-size limit quantity.

Let us test the effectiveness of the bound and its relevance to the problem of locating the SUSY transition in the WZ model.

For the quadratic potential $V = \lambda_2\varphi^2 + \lambda_0$, an argument by Witten suggest the existence of a negative number λ_0^* such that $\rho(\lambda_0)$ is positive when $\lambda_0 > \lambda_0^*$ and it vanishes for $\lambda_0 < \lambda_0^*$.

λ_0^* is the value of λ_0 in which dynamical SUSY breaking occurs.

Qualitative plot of the functions $\rho(\lambda_0)$ and $\rho^{(L)}(\lambda_0)$. We see that a single zero is expected in $\rho^{(L)}(\lambda_0)$ at some $\lambda_0 = \lambda_0(L)$. Since $\lim_{L\to\infty} \rho^{(L)} = \rho$, we expect that $\lambda_0(L) \to \lambda_0^*$ for $L \to \infty$ allowing for a determination of the critical coupling λ_0^*.

To obtain the numerical estimate of $\rho^{(L)}(\lambda_0)$ we used the worldline path integral (WLPI) algorithm.

The WLPI algorithm compute numerically the quantity

$$\rho^{(L)}(\beta, T) = \frac{1}{L} \frac{\mathrm{Tr}\{H\ (e^{-\frac{\beta}{T}H_1} e^{-\frac{\beta}{T}H_2})^T\}}{\mathrm{Tr}\{(e^{-\frac{\beta}{T}H_1} e^{-\frac{\beta}{T}H_2})^T\}}$$

The desired lower bound is obtained by the double extrapolation

$$\rho^{(L)} = \lim_{\beta\to\infty} \lim_{T\to\infty} \rho^{(L)}(\beta, T),$$

Numerically, we determined $\rho^{(L)}(\beta, T)$ for various values of β and T and a set of λ_0 that should include the transition point, at least according to the GFMC results.

Plot of the energy lower bound $\rho^{(L)}(\beta, T)$ at $L = 6$

Plot of the energy lower bound $\rho^{(L)}(\beta, T)$ at $L = 10$

Plot of the energy lower bound $\rho^{(L)}(\beta, T)$ at $L = 14$

Plot of the energy lower bound $\rho^{(L)}(\beta, T)$ at $L = 18$

Extrapolated bound $\rho^{(L)}(\beta) = \lim_{T\to\infty} \rho^{(L)}(\beta, T)$. As one can see, for each cluster size L, the two curves at the highest values of β almost coincide.

The best fit with a parabolic function gives $\lambda_0^* = -0.49 \pm 0.06$ quite in agreement with the previous $\lambda^*_{0,\mathrm{GFMC}} = -0.48 \pm 0.01$.

Plot of the energy lower bound $\rho^{(L)}(\beta, T)$ at various L, β and T for the cubic prepotential

Conclusions

- We studied the phase diagram of the lattice $N = 1$ Wess-Zumino model with a particular scalar potential for which we expect dynamical supersymmetry breaking.

- The key property of the formulation is the exact preservation of a SUSY subalgebra at finite lattice spacing.

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  - All the results for the model for a cubic prepotential indicate unbroken SUSY.
  - In the case of a quadratic prepotential we confirm the existence of two phases separated by a single phase transition at λ_0. This have been reported using two different methods.