Coupled waves in one-dimensional quasi-periodic structures, a Scilab toolbox project

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Abstract—A matrix-oriented, modular language like Scilab is suitable to realise a flexible tool for rapid evaluation of reflection, transmission, field distributions, laser gains, group delays, nonlinear effects for waves in one-dimensional quasi-periodic structures, using a coupled-mode analysis with a transfer matrix approach.

Keywords: Scilab, coupled waves, periodic structures, Bragg gratings, optical fibres, distributed feedback, photonic crystals, slow light.

I. INTRODUCTION

By “quasi-periodic” we mean a periodic structure (for ex. a dielectric, in the case of light) with - slow or sudden - phase or amplitude variations, or combinations of different periodic and uniform parts, and we study their properties for light (or any wave) having wavelength near the Bragg reflection peak. Such structures have been extensively studied from the 1970’s to recent times ([1-9] and hundreds of others), and used in devices based on passive or active optical fibres or on semiconductors, for ex. as Bragg reflectors, lambda/4-shifted laser cavities, chirped gratings, and devices enhancing nonlinear characteristics. More recently similar structures have been sometimes called “one-dimensional photonic crystals”. The concepts apply as well for any type of wave, as plasmons [10,11], acoustic waves, microwaves etc., and the periodic structures might be themselves waves [12], or due to nonlinear effects in the interference of waves (as in 4-wave mixing). Moreover, the subject - now rapidly developing - of “slow-light” applications is often based on quasi-periodic structures, as the strong chromatic dispersion near the resonant frequency allows, in some range, a very high group index [13].

In the one-dimensional case, in order to rapidly evaluate the characteristics of such structures as functions of various parameters, a modular and flexible tool like Scilab [14] is particularly suited. We have developed a (preliminary) toolbox which allows to calculate the transmission/reflection characteristics of waves going through an arbitrary assembly of quasi-periodic structures, the field distributions, including the effects of small nonlinearities. The method used is based on a transfer matrix approach [15,16], which is suited to languages like Scilab.

II. COUPLED MODE EQUATIONS AND COMPUTATION ALGORITHM

For a monochromatic wave, the (time-independent) field is approximated with two (forward and backward) counterpropagating modes F(z) and B(z):

\[ E(z) = F(z) \exp(-ibz) + B(z) \exp(ibz), \]

with propagation constants close to the Bragg condition:

\[ \beta \approx \pi/A, \] where \( \Lambda \) is a characteristic period of the material structure. The static (or quasi-static) quasi-periodic grating refractive index is assumed of the type

\[ \delta \epsilon = \epsilon_1(z) \cos(Qz + \phi(z)), \]

where \( \epsilon_1(z) \) and \( \phi(z) \), the grating amplitude and phase modulation are supposed to be either slowly variable with respect to the “carrier” \( \exp(iQz) \) or having points of discontinuity.

The equations describing the evolution along the position \( z \) of \( F(z) \) and \( B(z) \) can be written in the usual matrix form

\[
\frac{d}{dz} \begin{bmatrix} F(z) \\ B(z) \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} F(z) \\ B(z) \end{bmatrix}
\]

where the (hermitian) matrix \( M \) is of the type

\[
\begin{bmatrix} \sigma & -i \xi \\ i \bar{\xi}^c & \sigma \end{bmatrix}
\]

where \( \sigma \) is the propagation constant with the \( \exp(-ibz) \) factored out (\( = 0 \) if the structure is linear with no gain or absorption) and \( \xi(z) \) the coupling constant, including the phase factor \( \exp(i(\Delta \beta - \phi(z))) \)

If the structure has absorption or gain and is nonlinear, where \( g \) is the gain (<0 if absorbing), and \( \alpha \) the self phase modulation and \( \gamma \) the cross phase modulation

\[
\sigma = g/2 - i \alpha(|F|^2 + |B|^2)
\]

\[
\xi = \kappa(z) \exp(i(\Delta \beta - \phi(z))) + \gamma FB^c
\]
This form clearly lends itself to a Scilab implementation, with a matrix for each section of the structure. In some cases there is an analytic solution (as in a linear uniform section), in others a Runge-Kutta method is used, and in the nonlinear case an iterative solution is sought. The $F(0)=1$, $B(0)=0$ and the $F(0)=0$, $B(0)=1$ are mixed in order to satisfy the boundary conditions at $z=0$ and $z=L$ (end of the grating).

III. PROGRAM MODULES

The toolbox consists of a user interface made of modules: preliminary ones ("setup", and range of input wavelengths), the ones describing various types of gratings, followed by the command "compute", and commands for the display of data, and a computation backend made of compiled "C" functions dynamically linked to Scilab.

The script used to calculate a given structure appears like this:

- setup
- scan (range of wavelength deviations from Bragg condition)
- various sections: unif, vacuum, chirp,
- compute
- various plots: reflection, transmission, field distribution,

"scan.sci" defines the range of incoming wavevectors, expressed as detuning $\Delta \beta$ from half a (conventionally defined) Bragg wavevector $2\pi/\Lambda$: then $\Delta \beta=\beta-\pi/\Lambda$. A second argument can be a range of gain or absorption.

The "unif.sci" function, for uniform gratings, uses the analytic solution available in the linear case, the arguments are: the final value of $z$ (the initial one is the last one of the previous section, or zero), the coupling strength, the possible nonlinear coefficients, gain or loss, and the possible detuning from the given Bragg wavelength.

"vacuum.sci" means a section with no grating. Arguments are: final value of $z$, and optional gain/loss and third-order nonlinearity.

"chirp.sci" uses a Runge-Kutta algorithm and allows for an arbitrary amplitude and/or phase variation with $z$, as well as possible gain and/or nonlinearity. The amplitude/phase variation is entered as a string, a complex function $f(z)$ which is evaluated by the "eval" command.

"compute" may use an optional argument "f" if one wants to calculate the field distributions within the structure, and "g" for saving the modified grating in case of nonlinearity. In nonlinear cases an iterative solution is sought, using the compiled "iter2.c", and a warning appears in case of insufficient convergence.

The script then produces some vectors like "ref" (reflected amplitude), "tra" (transmitted amplitude), etc., which can be illustrated by plots. For ease of use, various plot modules are defined, allowing to show reflection and transmission as functions of detuning from the Bragg condition (sp_plot.sci), field distributions inside (f_plot.sci), group delays, “Q-factor” (q_plot.sci), group delay (gd_plot.sci), effects of absorption, gain or nonlinearity and reflection/transmission spectra (probe.sci) for a probe in the presence of a strong pump.

"C" compiled functions are essentially used in nonlinear cases, for the iteration procedure and other accessory procedures.

IV. SOME EXAMPLES

As any combination of (linear and nonlinear, with gain or loss) quasi-periodic gratings (uniform, with different centre wavelengths, amplitude and phase chirped, etc.) can be assembled and various parameters calculated, the number of possible examples is infinite; here we just show a few simple ones, for which the short script program is indicated.

1) A $\lambda/4$-shifted laser cavity: gain as function of detuning and of gain coefficient (showing in particular the peaks corresponding to oscillation conditions: it has to be noticed that sometimes an increase of gain quenches the oscillation). The command sequence is:

setup
detun=-6e-3:1e-4:6e-3;
gain=0:5e-6:5e-4;
scan(detun, gain)
unif(2e3,1e-3,0,0)
vacuum(8e3)
unif(1e4,-1e-3,0,0)
compute

and the resulting contour plot for abs(tra).^2 is:

![Figure 1: Distributed feedback laser gain as a function of gain coefficient and detuning, showing gain peaks (and then oscillation), which, on increasing gain coefficient (x-axis), moves away from the resonant wavelength (y-axis).](image)

2) A nonlinear grating, showing reflection spectrum change when the incoming wave is sufficiently strong:

setup
scan(-1e-2:0.5e-4:1e-2)
unif(2e3,1e-3,3e-4)
compute
sp_plot

(and repeat with unif(2e3,1e-3) to compare with linear case)
Figure 2: Reflection spectrum of a nonlinear grating (solid line), compared to the linear one, showing changes of nearly 80% at some wavelength near the peak.

3) The third example shows a simple uniform grating with or without a strong pump producing a 3rd order nonlinear effect (index variation proportional to intensity), showing reflection spectrum for a weak probe pump: the command sequence is:

```plaintext
setup
scan(-.01:2e-4:.01)
unif(1000,1e-3)
compute
sp_plot
setup
scan(0)
vacuum(2000,[-%i*2e-3,0])
unif(3000,1e-3)
compute('g')
probe(-0.01:2e-4:0.01)
pr_sp_plot('-.')
```

Figure 3: Narrowing of the reflectivity spectrum when a strong wave induces a transient grating in a nonlinear region in front of the grating.

4) Another example: the reflection of a wave on a nonlinear grating, with increasing and decreasing intensity, showing a hysteresis effect:

```plaintext
setup
scan(1e-3)
unif(2000,1e-3,.66e-3)
hyst(3000)  //increase/decrease intensity in 3000 steps
hy_plot
```

Figure 4: Reflectivity of a strong wave from a nonlinear grating, as a function of intensity (arbitrary units), showing a hysteresis effect.

5) The group delay in transmission is proportional to \( \text{diff} (\text{atan} (\text{real}(\text{tra}), \text{imag}(\text{tra}))) \). It might be necessary to use a function to unwrap the phase (particularly in reflection). This example shows the very high group delay change near the resonant wavelength, obtainable with a \( \lambda/4 \)-shifted structure.

Figure 5: Strong variation near resonance of transmission group delay in two cascaded uniform gratings with a \( \pi \) shift (solid line, arbitrary units), compared with a uniform grating of the same total length.

V. MULTIPLE WAVEGUIDES

The program described above calculates the interaction of contra-propagating waves in one waveguide. An independent module has been written in order to describe multiple waveguides coupled by a co-propagating coupling constant, with contra-propagating coupling due to Bragg reflectors. This is limited to the linear case. At present the module is written for two waveguides, (with four waves \( F_1, F_2, B_1 \) and \( B_2 \), with co-directional coupling \( \alpha \) and contra-directional coupling \( \kappa \)), but can be easily generalised to more [17,18]. The equations for the two forward and backwards amplitudes are:

\[
\frac{dF_1}{dz} = \alpha F_2 + \kappa B_1 \quad ; \quad \frac{dB_1}{dz} = -\alpha B_2 + \kappa F_1
\]

\[
\frac{dF_2}{dz} = -\alpha F_1 + \kappa B_2 \quad ; \quad \frac{dB_2}{dz} = \alpha B_1 + \kappa F_2
\]

An example of output shows the 4 outputs for two coupled waveguides, both with a Bragg grating.
A simple module has also been written for N waveguides with only co-directional couplings.

VI. Conclusions

An ensemble of Scilab “.sci” functions and compiled “C” functions have been assembled in order to calculate the characteristics of any sequence of quasi-periodic Bragg reflectors, both linear and nonlinear. For the linear case, this has been extended to multiple, coupled forward and backwards waves. Some bugs still have to be fixed, and many extensions can be developed. The programs can be obtained from the authors, and will be submitted to Scilab as a toolbox project.

References